Differentiation - Local or Relative Min/Max

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Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

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Question 1

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Differentiation

Subtopics: Vertical Tangents, Local or Relative Minima and Maxima, Concavity, Tangents To Curves, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 4

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

Question 2

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Differentiation

Subtopics: Points Of Inflection, Local or Relative Minima and Maxima, Integration Technique – Standard Functions, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Medium / Question Number: 5

5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a, b, and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

(a) Find the values of a and b.

(b) If $\int_0^1 f(x)\,dx = 32$, what is the value of k ?

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Question 3

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Local or Relative Minima and Maxima, Particular Solution of Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Very Hard / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

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Question 4

Qualification: AP Calculus AB

Areas: Differentiation, Applications of Differentiation

Subtopics: Implicit Differentiation, Tangents To Curves, Local or Relative Minima and Maxima

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Very Hard / Question Number: 4

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.

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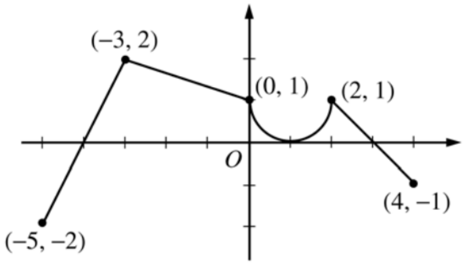
Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Derivative Graphs, Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Points Of Inflection, Integration Graphs

Paper: Part B-Non-Calc / Series: 2004 / Difficulty: Somewhat Challenging / Question Number: 5



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

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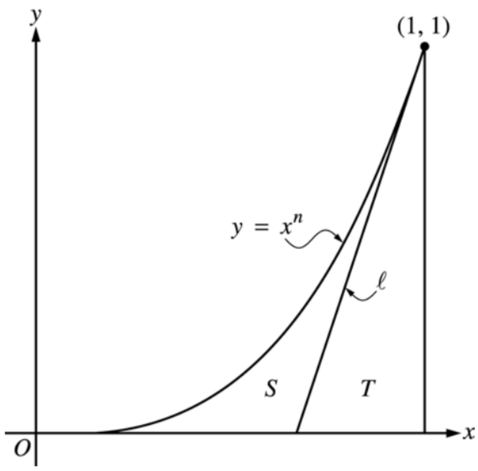
Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Standard Functions, Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2004-Form-B / Difficulty: Hard / Question Number: 6



6. Let ℓ be the line tangent to the graph of $y = x^n$ at the point $(1, 1)$, where $n > 1$, as shown above.

(a) Find $\int_0^1 x^n \, dx$ in terms of n.

(b) Let T be the triangular region bounded by ℓ, the x-axis, and the line $x = 1$. Show that the area of T is $\frac{1}{2n}$.

(c) Let S be the region bounded by the graph of $y = x^n$, the line ℓ, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Derivative Tables, Derivative Graphs, Fundamental Theorem of Calculus (Second), Points Of Inflection, Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Somewhat Challenging / Question Number: 4

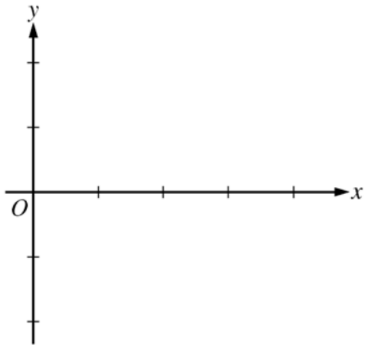
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

(Note: Use the axes provided in the pink test booklet.)



(c) Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.

(d) For the function g defined in part (c), find all values of x, for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

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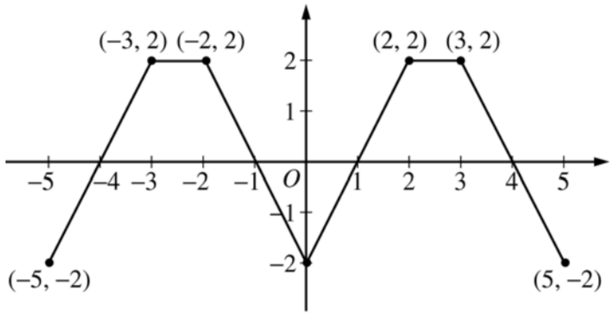
Question 8

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Derivative Graphs, Fundamental Theorem of Calculus (Second), Local or Relative Minima and Maxima, Tangents To Curves

Paper: Part A-Calc / Series: 2006 / Difficulty: Hard / Question Number: 3



Graph of f

3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(4)$, $g'(4)$, and $g''(4)$.

(b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.

(c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f. Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.

Question 9

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Differentiation Technique – Standard Functions, Local or Relative Minima and Maxima, Points Of Inflection

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Easy / Question Number: 6

6. Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

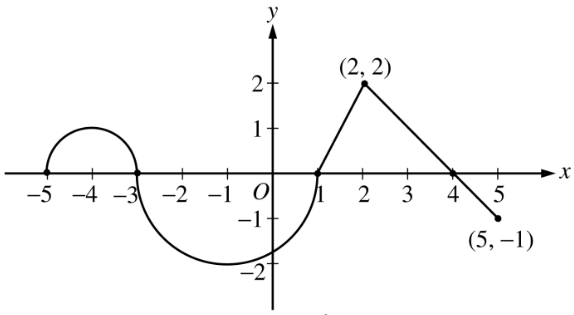
Question 10

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Points Of Inflection, Concavity, Increasing/Decreasing , Derivative Graphs, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Easy / Question Number: 4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

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Question 11

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

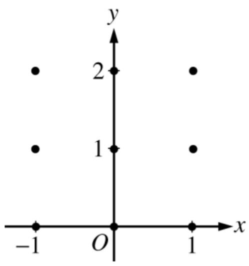
Subtopics: Sketching Slope Field, Concavity, Initial Conditions in Differential Equation, Local or Relative Minima and Maxima, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

Question 12

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Rates of Change (Average), Riemann Sums – Trapezoidal Rule, Mean Value Theorem, Intermediate Value Theorem, Local or Relative Minima and Maxima, Total Amount

Paper: Part A-Calc / Series: 2008 / Difficulty: Hard / Question Number: 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

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Question 13

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Limits and Continuity

Subtopics: Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule, Points Of Inflection, Calculating Limits Algebraically, Tangents To Curves

Paper: Part B-Non-Calc / Series: 2008 / Difficulty: Easy / Question Number: 6

6. Let f be the function given by $f(x) = \dfrac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \dfrac{1 - \ln x}{x^2}$.

(a) Write an equation for the line tangent to the graph of f at $x = e^2$.

(b) Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

(c) The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

(d) Find $\lim\limits_{x \to 0^+} f(x)$.

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Question 14

Qualification: AP Calculus AB

Areas: Applications of Integration, Applications of Differentiation

Subtopics: Total Amount, Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (First), Average Value of a Function

Paper: Part A-Calc / Series: 2009 / Difficulty: Easy / Question Number: 2

2. The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

Question 15

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Modelling Situations, Interpreting Meaning in Applied Contexts, Fundamental Theorem of Calculus (Second), Global or Absolute Minima and Maxima, Local or Relative Minima and Maxima

Paper: Part A-Calc / Series: 2009 / Difficulty: Somewhat Challenging / Question Number: 3

3. Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is $6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

(a) Find Mighty's profit on the sale of a 25-meter cable.

(b) Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}\,dx$ in the context of this problem.

(c) Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is k meters long.

(d) Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

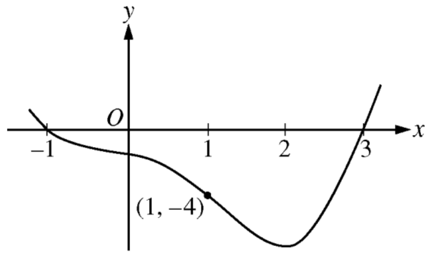
Question 16

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Tangents To Curves, Local or Relative Minima and Maxima, Increasing/Decreasing , Rates of Change (Average), Derivative Graphs

Paper: Part B-Non-Calc / Series: 2009-Form-B / Difficulty: Hard / Question Number: 5



Graph of f'

5. Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

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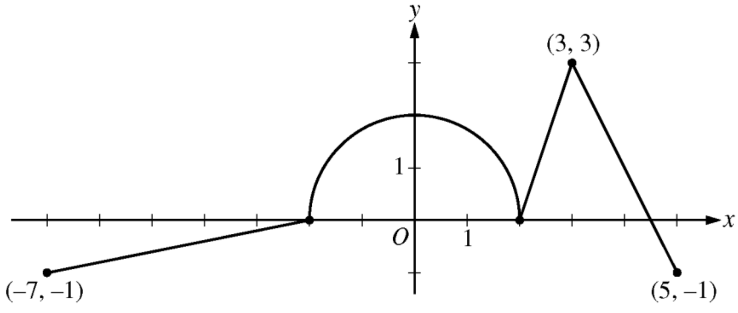
Question 17

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation

Subtopics: Derivative Graphs, Integration Technique – Geometric Areas, Points Of Inflection, Local or Relative Minima and Maxima

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Somewhat Challenging / Question Number: 5



Graph of g'

5. The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

(a) Find $g(3)$ and $g(-2)$.

(b) Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.

(c) The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

Question 18

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Concavity, Differentiation Technique – Standard Functions, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Medium / Question Number: 4

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

   (a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

   (b) Find all intervals on which the graph of f is concave down. Justify your answer.

   (c) Given that $f(1) = 2$, determine the function f.

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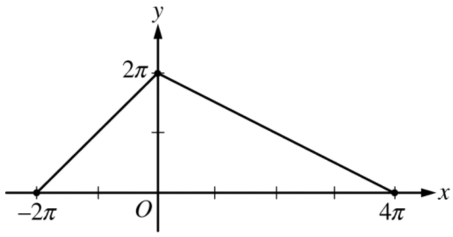
Question 19

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Differentiation Technique – Trigonometry, Fundamental Theorem of Calculus (Second)

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Somewhat Challenging / Question Number: 6



Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_{0}^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

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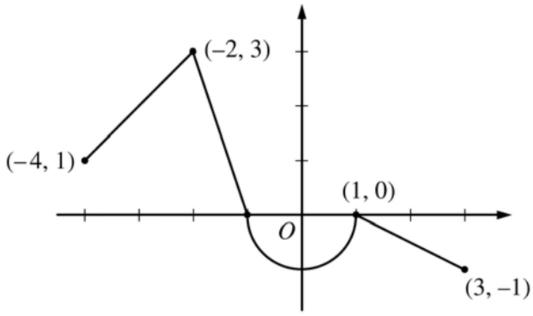
Question 20

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Fundamental Theorem of Calculus (Second), Local or Relative Minima and Maxima, Points Of Inflection, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Medium / Question Number: 3



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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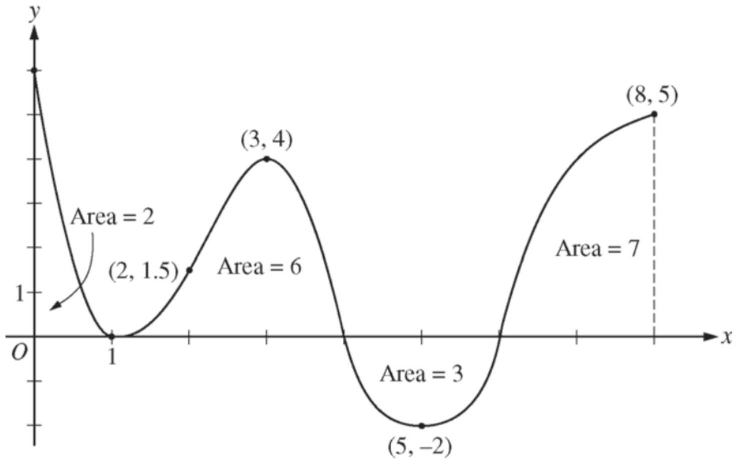
Question 21

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Concavity, Increasing/Decreasing , Implicit Differentiation, Tangents To Curves, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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Question 22

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Mean Value Theorem, Local or Relative Minima and Maxima, Derivative Tables, Differentiation Technique – Chain Rule, Fundamental Theorem of Calculus (First)

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Somewhat Challenging / Question Number: 5

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

5. The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

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Question 23

Qualification: AP Calculus AB

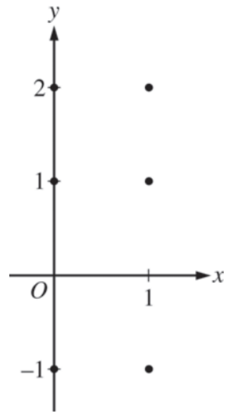
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Concavity, Local or Relative Minima and Maxima, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Medium / Question Number: 4

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

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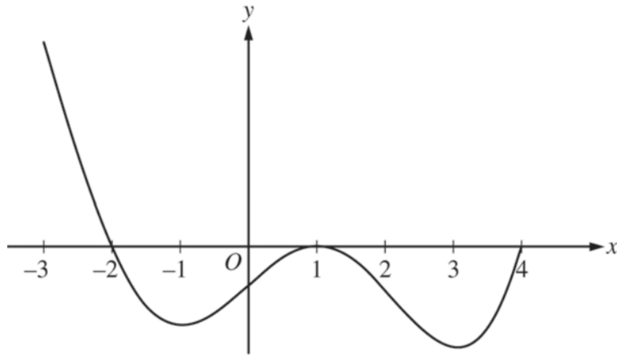
Question 24

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Increasing/Decreasing , Concavity, Points Of Inflection, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Easy / Question Number: 5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

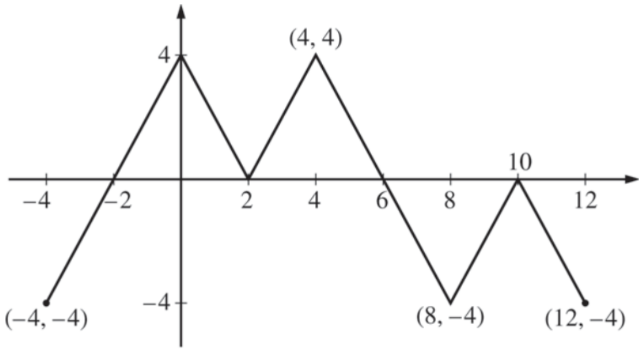
Question 25

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Points Of Inflection, Global or Absolute Minima and Maxima, Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2016 / Difficulty: Medium / Question Number: 3



Graph of f

3. The figure above shows the graph of the piecewise-linear function f. For $-4 \le x \le 12$, the function g is defined by $g(x) = \int_2^x f(t)\,dt$.

(a) Does g have a relative minimum, a relative maximum, or neither at $x = 10$? Justify your answer.

(b) Does the graph of g have a point of inflection at $x = 4$? Justify your answer.

(c) Find the absolute minimum value and the absolute maximum value of g on the interval $-4 \le x \le 12$. Justify your answers.

(d) For $-4 \le x \le 12$, find all intervals for which $g(x) \le 0$.

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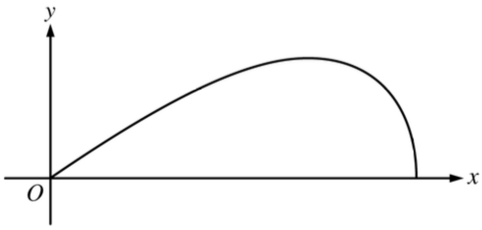
Question 26

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Integration

Subtopics: Integration - Area Under A Curve, Integration Technique – Substitution, Local or Relative Minima and Maxima, Volume of Revolution – Disc Method

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Medium / Question Number: 3



3. A company designs spinning toys using the family of functions $y = cx\sqrt{4 - x^2}$, where c is a positive constant. The figure above shows the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$, for some c. Each spinning toy is in the shape of the solid generated when such a region is revolved about the x-axis. Both x and y are measured in inches.

(a) Find the area of the region in the first quadrant bounded by the x-axis and the graph of $y = cx\sqrt{4 - x^2}$ for $c = 6$.

(b) It is known that, for $y = cx\sqrt{4 - x^2}$, $\dfrac{dy}{dx} = \dfrac{c\left(4 - 2x^2\right)}{\sqrt{4 - x^2}}$. For a particular spinning toy, the radius of the largest cross-sectional circular slice is 1.2 inches. What is the value of c for this spinning toy?

(c) For another spinning toy, the volume is 2π cubic inches. What is the value of c for this spinning toy?

Question 27

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Implicit Differentiation, Differentiation Technique – Trigonometry, Differentiation Technique – Product Rule, Tangents To Curves, Local or Relative Minima and Maxima, Differentiation Technique - Quotient Rule

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Hard / Question Number: 5

5. Consider the function $y = f(x)$ whose curve is given by the equation $2y^2 - 6 = y \sin x$ for $y > 0$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{y \cos x}{4y - \sin x}$.

(b) Write an equation for the line tangent to the curve at the point $(0, \sqrt{3})$.

(c) For $0 \leq x \leq \pi$ and $y > 0$, find the coordinates of the point where the line tangent to the curve is horizontal.

(d) Determine whether f has a relative minimum, a relative maximum, or neither at the point found in part (c). Justify your answer.

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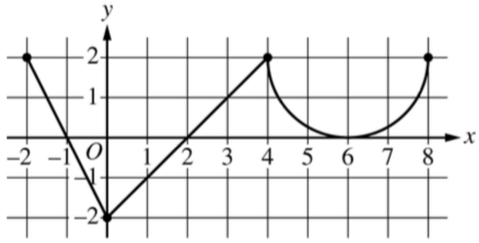
Question 28

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Concavity, Derivative Graphs, Global or Absolute Minima and Maxima, Integration Technique – Geometric Areas, L'Hôpital's Rule, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Medium / Question Number: 4



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim\limits_{x \to 2} \dfrac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

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Question 29

Qualification: AP Calculus AB

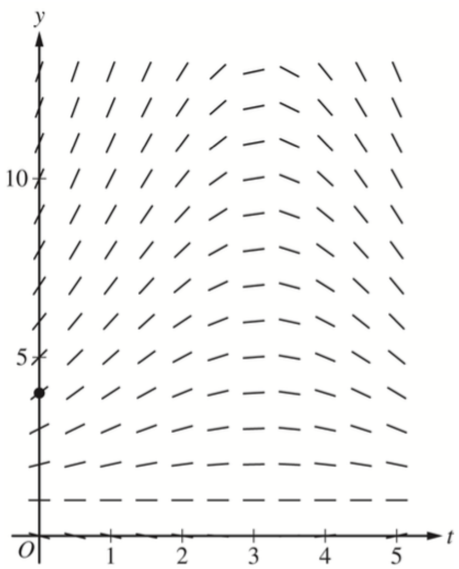
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Local or Relative Minima and Maxima, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation, Integration Technique – Trigonometry

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 3

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

   (a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



   (b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

   (c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

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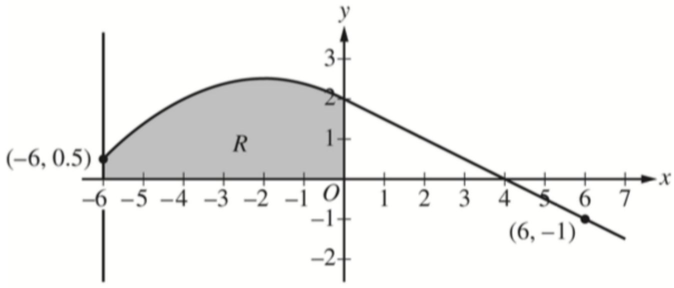
Question 30

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Fundamental Theorem of Calculus (Second), Derivative Graphs, Integration Graphs

Paper: Part B-Non-Calc / Series: 2024 / Difficulty: Medium / Question Number: 4



Graph of f

4. The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

(a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

(c) The function h is defined by $h(x) = \int_{-6}^x f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

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